

Question 2

Evaluate

$$\iint_R e^{2x} \cos y \, dA, \quad R = [0, \ln 3] \times \left[0, \frac{\pi}{3}\right].$$

Tasks

1. Recognize and use the product structure of the integrand.
2. Evaluate the integral in each order.
3. Verify the exact result by multiplying the two one-variable integrals.

Question 2 – Solution

Strategy. The integrand is a product of an x -only factor and a y -only factor, so constant rectangular bounds make the two integrations separate.

Step 1: Integrate in y first

$$\begin{aligned}\int_0^{\ln 3} \int_0^{\pi/3} e^{2x} \cos y \, dy \, dx &= \int_0^{\ln 3} e^{2x} [\sin y]_0^{\pi/3} \, dx \\ &= \frac{\sqrt{3}}{2} \left[\frac{1}{2} e^{2x} \right]_0^{\ln 3} \\ &= \frac{\sqrt{3}}{4} (9 - 1) = 2\sqrt{3}.\end{aligned}$$

Step 2: Reverse the order

$$\begin{aligned}\int_0^{\pi/3} \int_0^{\ln 3} e^{2x} \cos y \, dx \, dy &= \int_0^{\pi/3} \cos y \left[\frac{1}{2} e^{2x} \right]_0^{\ln 3} \, dy \\ &= 4[\sin y]_0^{\pi/3} = 2\sqrt{3}.\end{aligned}$$

Therefore

$$\boxed{\iint_R e^{2x} \cos y \, dA = 2\sqrt{3}}.$$

Verification Direct factorization gives

$$\left(\int_0^{\ln 3} e^{2x} \, dx \right) \left(\int_0^{\pi/3} \cos y \, dy \right) = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3},$$

confirming both orders and the endpoint $e^{2\ln 3} = 9$.