

Question 1

Evaluate the double integral of

$$f(x, y) = 3x^2y - 2x + 4$$

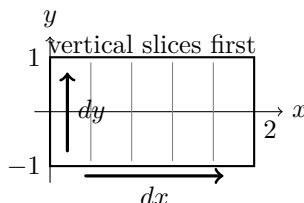
over $R = [0, 2] \times [-1, 1]$ by iterated integration.

Tasks

1. Evaluate by integrating with respect to y first.
2. Evaluate again by integrating with respect to x first.
3. Explain why the two answers must agree.

Question 1 – Solution

Strategy. Write both constant-bound orders explicitly and simplify the inner integral before performing the outer one.



Step 1: Integrate in y first

$$\begin{aligned}\int_0^2 \int_{-1}^1 (3x^2y - 2x + 4) dy dx &= \int_0^2 \left[\frac{3}{2}x^2y^2 - 2xy + 4y \right]_{-1}^1 dx \\ &= \int_0^2 (-4x + 8) dx \\ &= [-2x^2 + 8x]_0^2 = 8.\end{aligned}$$

The $3x^2y$ term vanishes because it is odd in y over a symmetric interval.

Step 2: Integrate in x first

$$\begin{aligned}\int_{-1}^1 \int_0^2 (3x^2y - 2x + 4) dx dy &= \int_{-1}^1 [x^3y - x^2 + 4x]_0^2 dy \\ &= \int_{-1}^1 (8y + 4) dy = 8.\end{aligned}$$

Thus

$$\boxed{\iint_R f dA = 8}.$$

Verification The polynomial is continuous on the closed rectangle, so Fubini's theorem permits either order. In the second order the odd term $8y$ cancels, leaving constant height 4 across a y -interval of length 2, again giving 8.