

Question 10

A sphere of radius R has surface-area-to-volume ratio

$$\frac{S}{V} = \frac{3}{2}.$$

Determine its radius, surface area, and volume.

Tasks

1. Derive V with a spherical-coordinate triple integral.
2. Derive S by viewing the two hemispheres as graphs.
3. Use the given ratio and verify its units.

Question 10 – Solution

Strategy. Rebuild both standard sphere formulas from integration, then use their ratio to recover the radius.

Step 1: Volume In spherical coordinates,

$$V = \int_0^{2\pi} \int_0^\pi \int_0^R \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = 2\pi(2) \frac{R^3}{3} = \frac{4\pi R^3}{3}.$$

Step 2: Surface area For the upper graph $z = \sqrt{R^2 - r^2}$, the area factor is $R/\sqrt{R^2 - r^2}$. Its area is

$$S_{\text{upper}} = \int_0^{2\pi} \int_0^R \frac{Rr}{\sqrt{R^2 - r^2}} \, dr \, d\theta = 2\pi R^2.$$

Doubling for both hemispheres gives $S = 4\pi R^2$.

Step 3: Use the ratio

$$\frac{S}{V} = \frac{4\pi R^2}{4\pi R^3/3} = \frac{3}{R} = \frac{3}{2}, \quad \boxed{R = 2}.$$

Therefore

$$\boxed{S = 16\pi}, \quad \boxed{V = \frac{32\pi}{3}}.$$

Verification Surface area divided by volume has units of inverse length, and $3/R$ has those units. Substituting $R = 2$ reproduces the stated ratio.