

Question 8

For $A > 0$, let E_A be the solid bounded below by

$$z = x^2 + y^2$$

and above by the plane $z = A$. If E_A has volume 18π , determine A , the base radius, and the height.

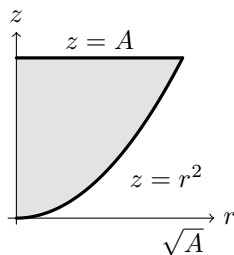
Tasks

1. Derive a formula for $V(A)$.
2. Solve the inverse volume condition.
3. Verify the dimensions and positivity of the result.

Question 8 – Solution

Strategy. First express the volume as a function of the unknown cutting height A , then solve the resulting equation.

Step 1: Geometry and volume



The plane and paraboloid meet when $r^2 = A$, so $0 \leq r \leq \sqrt{A}$. Thus

$$\begin{aligned} V(A) &= \int_0^{2\pi} \int_0^{\sqrt{A}} (A - r^2)r \, dr \, d\theta \\ &= 2\pi \left[\frac{Ar^2}{2} - \frac{r^4}{4} \right]_0^{\sqrt{A}} = \frac{\pi A^2}{2}. \end{aligned}$$

Step 2: Recover the dimensions The condition $V(A) = 18\pi$ yields

$$\frac{\pi A^2}{2} = 18\pi, \quad A^2 = 36.$$

Since $A > 0$,

$$\boxed{A = 6}, \quad \boxed{\text{base radius} = \sqrt{6}}, \quad \boxed{\text{height} = 6}.$$

Verification Substitution gives $V(6) = \pi(36)/2 = 18\pi$. The positive root is required because A is a height, and $r = \sqrt{A}$ follows from the intersection equation.