

**Question 3**

Find the volume enclosed between the paraboloids

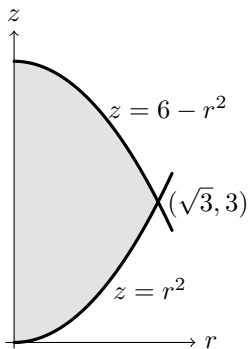
$$z = x^2 + y^2 \quad \text{and} \quad z = 6 - x^2 - y^2.$$

**Tasks**

1. Find the curve where the surfaces meet.
2. Set up and evaluate a cylindrical-coordinate integral.
3. Check that the height is nonnegative throughout the base.

**Question 3 – Solution**

**Strategy.** Subtract the lower paraboloid from the upper one over their circular projection.

**Step 1: Intersection**

Setting the surfaces equal gives

$$r^2 = 6 - r^2, \quad r = \sqrt{3}, \quad z = 3.$$

Hence the projection is the disk  $0 \leq r \leq \sqrt{3}$ .

**Step 2: Volume** The vertical height is  $(6 - r^2) - r^2 = 6 - 2r^2$ . Therefore

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^{\sqrt{3}} (6 - 2r^2)r \, dr \, d\theta \\ &= 2\pi \left[ 3r^2 - \frac{r^4}{2} \right]_0^{\sqrt{3}} = 2\pi \left( 9 - \frac{9}{2} \right) = \boxed{9\pi}. \end{aligned}$$

**Verification** On  $0 \leq r \leq \sqrt{3}$ , the height is  $2(3 - r^2) \geq 0$ ; it decreases from 6 at the axis to 0 at the intersection circle, matching the geometry.