

Question 2

Find the area common to the disks

$$x^2 + y^2 \leq 2x, \quad x^2 + y^2 \leq 2y.$$

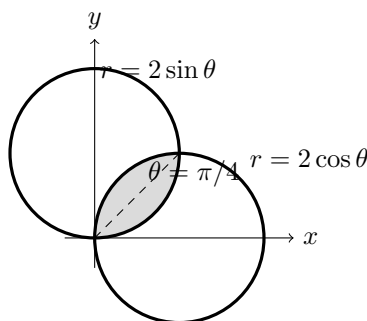
Tasks

1. Translate both inequalities into polar coordinates.
2. Identify where the radial boundary changes and evaluate the area.
3. Verify the symmetry used in the calculation.

Question 2 – Solution

Strategy. In the first quadrant the smaller of the two polar bounds switches at the line $\theta = \pi/4$.

Step 1: Polar description



For $r \geq 0$, the disks become

$$r \leq 2 \cos \theta, \quad r \leq 2 \sin \theta.$$

Their common region lies in $0 \leq \theta \leq \pi/2$. The bounds are equal at $\theta = \pi/4$.

Step 2: Evaluate Reflection across $y = x$ makes the two halves congruent, so

$$\begin{aligned} A &= 2 \int_0^{\pi/4} \int_0^{2 \sin \theta} r \, dr \, d\theta = 4 \int_0^{\pi/4} \sin^2 \theta \, d\theta \\ &= 4 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/4} = \boxed{\frac{\pi}{2} - 1}. \end{aligned}$$

Verification The circles have equal radius and centers $(1, 0)$ and $(0, 1)$, which are interchanged by reflecting across $y = x$. Their intersections, $(0, 0)$ and $(1, 1)$, also lie on that line.