

Question 7

The plane $z = 6 + x - 2y$ lies above the rectangle

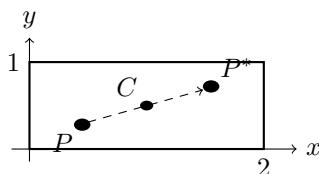
$$R = [0, 2] \times [0, 1].$$

Tasks

1. Pair each point of R with its reflection through the center $(1, 1/2)$ and compare their heights.
2. Use the pairing to find the volume under the plane without iterated integration.
3. Verify the result using the four corner heights.

Question 7 – Solution

Strategy. An affine function has complementary values at points reflected through a rectangle's center, so every reflected pair has the same average height.



Step 1: Reflected heights The reflection of $P = (x, y)$ through $C = (1, 1/2)$ is

$$P^* = (2 - x, 1 - y).$$

For $f(x, y) = 6 + x - 2y$,

$$\begin{aligned} f(P) + f(P^*) &= (6 + x - 2y) + (6 + (2 - x) - 2(1 - y)) \\ &= 12. \end{aligned}$$

Thus every pair has average height $6 = f(C)$.

Step 2: Volume The base area is 2, and the plane's minimum corner height is $4 > 0$, so the double integral is geometric volume:

$$V = \iint_R (6 + x - 2y) dA = 6(2) = 12.$$

Step 3: Corner check The corner heights are

$$f(0, 0) = 6, \quad f(2, 0) = 8, \quad f(0, 1) = 4, \quad f(2, 1) = 6.$$

Their average is $(6 + 8 + 4 + 6)/4 = 6$, agreeing with the center and pairing calculation. Multiplication by base area again gives 12.