

Question 5

Let $R = [-1, 2] \times [0, 2]$. A continuous function f satisfies

$$2 \leq f(x, y) \leq 5, \quad \iint_R f \, dA = 19.$$

A second integrable function g satisfies $|g(x, y) - f(x, y)| \leq 0.1$ throughout R .

Tasks

1. Check that the stated integral of f is consistent with its pointwise bounds.
2. Evaluate $\iint_R (3f - 4) \, dA$.
3. Give the sharp bounds guaranteed by the data for $\iint_R g \, dA$.

Question 5 – Solution

Strategy. Convert pointwise inequalities to integral inequalities using the area of the rectangle, and use linearity for the transformed function.

Step 1: Comparison bounds The rectangle has area

$$A = (2 - (-1))(2 - 0) = 6.$$

Integrating $2 \leq f \leq 5$ over R gives

$$12 \leq \iint_R f \, dA \leq 30.$$

The given value 19 lies in this interval, so the data are consistent.

Step 2: Linearity Because the integral of the constant 4 is $4A = 24$,

$$\iint_R (3f - 4) \, dA = 3 \iint_R f \, dA - \iint_R 4 \, dA = 57 - 24 = \boxed{33}.$$

Step 3: Stability bound From $|g - f| \leq 0.1$,

$$-0.1 \leq g - f \leq 0.1.$$

Integration over an area-6 region yields

$$-0.6 \leq \iint_R (g - f) \, dA \leq 0.6.$$

Consequently,

$$\boxed{18.4 \leq \iint_R g \, dA \leq 19.6}.$$

These endpoints are attainable by $g = f - 0.1$ and $g = f + 0.1$, so no tighter bounds follow from the supplied information.