

Question 4

On $R = [-2, 2] \times [0, 3]$, define

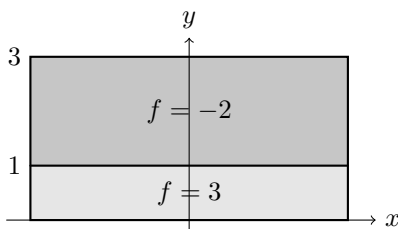
$$f(x, y) = \begin{cases} 3, & 0 \leq y \leq 1, \\ -2, & 1 < y \leq 3. \end{cases}$$

Tasks

1. Evaluate $\iint_R f \, dA$ by signed volume.
2. Evaluate $\iint_R |f| \, dA$ and explain the difference.
3. Find the average value of f on R .

Question 4 – Solution

Strategy. Split the base into the two constant-height bands and distinguish algebraic volume from total unsigned volume.



Step 1: Signed volume The lower band has area $4(1) = 4$ and height 3, so it contributes 12. The upper band has area $4(2) = 8$ and height -2 , so it contributes -16 . Hence

$$\iint_R f \, dA = 12 - 16 = -4.$$

Step 2: Unsigned volume Absolute value changes the second height from -2 to 2. Therefore

$$\iint_R |f| \, dA = 12 + 16 = 28.$$

The first integral records volume below the xy -plane negatively; the second records all geometric volume positively.

Step 3: Average value Since $\text{area}(R) = 4 \cdot 3 = 12$,

$$f_{\text{avg}} = \frac{1}{12} \iint_R f \, dA = -\frac{1}{3}.$$

As a check, $-1/3$ lies between the only two function values, -2 and 3 , and the wider negative band explains its negative sign.