

Question 1

Let

$$R = [0, 2] \times [0, 1], \quad f(x, y) = 6 - x^2 - y.$$

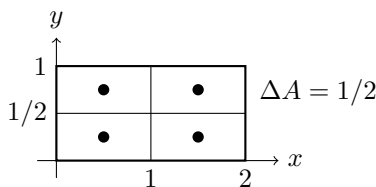
Partition R into two equal columns and two equal rows, and use the midpoint of each subrectangle as its sample point.

Tasks

1. List the four sample points and the common subrectangle area.
2. Compute the midpoint Riemann sum.
3. Interpret the sum geometrically and check it against bounds from the minimum and maximum of f on R .

Question 1 – Solution

Strategy. Translate the partition into four base areas, evaluate the height at each midpoint, and add the resulting prism volumes.



Step 1: Partition data Here $\Delta x = 1$, $\Delta y = 1/2$, and

$$\Delta A = \Delta x \Delta y = \frac{1}{2}.$$

The midpoints are

$$\left(\frac{1}{2}, \frac{1}{4}\right), \left(\frac{3}{2}, \frac{1}{4}\right), \left(\frac{1}{2}, \frac{3}{4}\right), \left(\frac{3}{2}, \frac{3}{4}\right).$$

Step 2: Heights and sum In the same order, the four function values are

$$\frac{11}{2}, \quad \frac{7}{2}, \quad 5, \quad 3.$$

Therefore

$$S = \frac{1}{2} \left(\frac{11}{2} + \frac{7}{2} + 5 + 3 \right) = \boxed{\frac{17}{2}}.$$

Step 3: Interpretation and bounds The sum is the total volume of four midpoint prisms. On R , f decreases as either x or y increases, so

$$1 = f(2, 1) \leq f(x, y) \leq f(0, 0) = 6.$$

Since $\text{area}(R) = 2$, every Riemann-sum estimate lies between 2 and 12; indeed $17/2$ does. This also verifies that every sampled height was positive and on the correct scale.