

Question 9

A firm must choose positive inputs x, y satisfying

$$xy = 100.$$

Its cost is

$$C(x, y) = 3x + 2y.$$

Tasks

1. Use Lagrange multipliers to find the cost-minimizing inputs.
2. Compute the minimum cost and interpret the multiplier balance.
3. Verify global minimality by substitution or an inequality.

Question 9 – Solution

Strategy. The multiplier equations balance marginal costs against the constraint normal; then a one-variable check rules out other behavior.

Step 1: Multiplier equations With $g = xy$,

$$\langle 3, 2 \rangle = \lambda \langle y, x \rangle, \quad xy = 100.$$

Thus $3 = \lambda y$ and $2 = \lambda x$. Eliminating λ gives

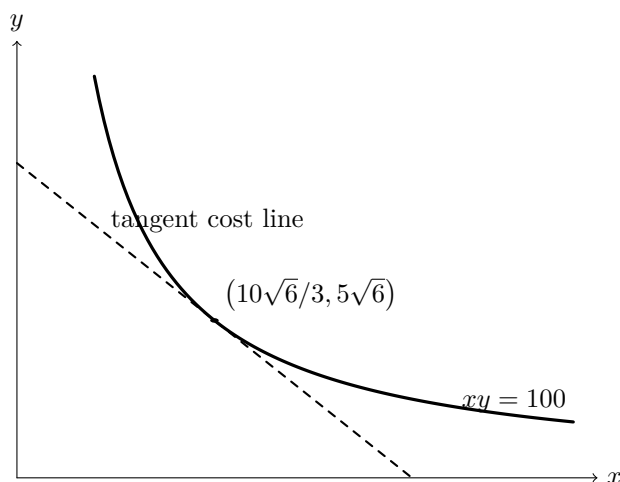
$$3x = 2y, \quad y = \frac{3}{2}x.$$

The positive constraint then yields

$$x^2 = \frac{200}{3},$$

so

$$x = \frac{10\sqrt{6}}{3}, \quad y = 5\sqrt{6}.$$



Step 2: Cost and interpretation

$$C_{\min} = 3 \left(\frac{10\sqrt{6}}{3} \right) + 2(5\sqrt{6}) = 20\sqrt{6}.$$

The relation $3/y = 2/x = \lambda$ says the cost-gradient components are proportional to the production-constraint gradient components.

Step 3: Global verification Using $y = 100/x$ for $x > 0$,

$$C(x) = 3x + \frac{200}{x} \longrightarrow \infty$$

as $x \rightarrow 0^+$ or $x \rightarrow \infty$. Its only critical point is the one found above, and $C''(x) = 400/x^3 > 0$. Hence it is the unique absolute constrained minimum.