

Question 6

Consider maximizing and minimizing

$$f(x, y) = x$$

subject to the constraint

$$g(x, y) = x^2 + y^2 = 0.$$

Tasks

1. Determine the feasible set and its constrained extrema directly.
2. Show that no multiplier λ satisfies $\nabla f = \lambda \nabla g$ at the feasible point.
3. Explain which constraint-qualification hypothesis fails and why this does not contradict the method.

Question 6 – Solution

Strategy. Inspect the feasible set before applying the multiplier equation; the constraint has a singular gradient at its only point.

Step 1: Direct analysis The equation $x^2 + y^2 = 0$ over the reals forces

$$x = 0, \quad y = 0.$$

Thus the feasible set is the singleton $\{(0, 0)\}$. Its only function value is 0, so

$$\boxed{f_{\min} = f_{\max} = 0 \text{ at } (0, 0)}.$$

Step 2: Multiplier failure

$$\nabla f(0, 0) = \langle 1, 0 \rangle, \quad \nabla g(0, 0) = \langle 0, 0 \rangle.$$

The equation

$$\langle 1, 0 \rangle = \lambda \langle 0, 0 \rangle$$

has no solution for any real λ .

Step 3: Interpretation The standard Lagrange multiplier theorem assumes $\nabla g \neq \mathbf{0}$ at the constrained extremum. That regularity condition fails here, so the theorem makes no promise. The example shows that singular feasible points must be checked separately; it does not invalidate the multiplier method at regular constraint points.