

Question 1

Find the constrained extrema of

$$f(x, y) = x + y$$

subject to

$$x^2 + y^2 = 1.$$

Tasks

1. Write and solve the Lagrange multiplier equations.
2. Determine the maximum and minimum values and their locations.
3. Interpret the multiplier condition as tangency of level curves.

Question 1 – Solution

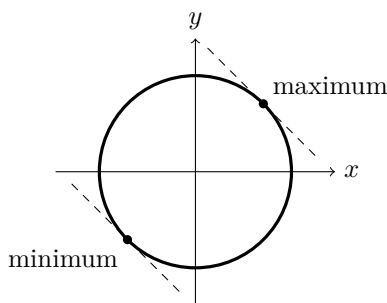
Strategy. Solve $\nabla f = \lambda \nabla g$ together with the circular constraint, then compare the finite candidate set.

Step 1: Lagrange system Let $g = x^2 + y^2$. Then

$$\langle 1, 1 \rangle = \lambda \langle 2x, 2y \rangle, \quad x^2 + y^2 = 1.$$

The first two equations imply $x = y$. Hence $2x^2 = 1$, so

$$(x, y) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \quad \text{or} \quad \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right).$$



Step 2: Values Direct evaluation gives

$$f_{\max} = \sqrt{2} \text{ at } \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right),$$

$$f_{\min} = -\sqrt{2} \text{ at } \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right).$$

Step 3: Geometry and verification The level curves $x + y = c$ are parallel lines with normal ∇f . At an extreme one such line is tangent to the circle, so its normal is parallel to the radial normal ∇g . The circle is compact and the two candidates exhaust the multiplier system, completing the verification.