

Question 10

Find the absolute extrema of the nondifferentiable function

$$f(x, y) = |x| + y^2$$

on the closed unit disk $x^2 + y^2 \leq 1$.

Tasks

1. Analyze the interior, including the crease $x = 0$ where f_x does not exist.
2. Reduce the circular boundary problem to a single variable $u = |x|$.
3. Find and verify all absolute minimizing and maximizing points.

Question 10 – Solution

Strategy. A complete candidate search must include nondifferentiable points. On the circle, replace y^2 by $1 - x^2$.

Step 1: Interior and crease Where $x \neq 0$,

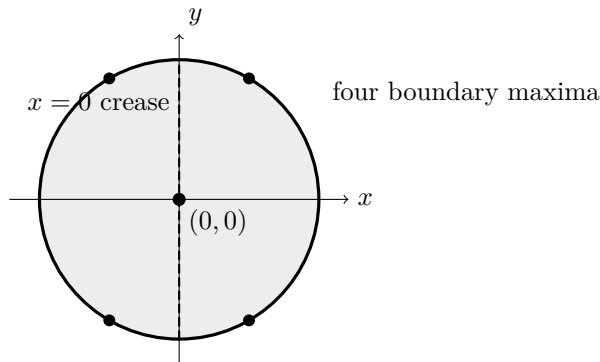
$$f_x = \operatorname{sgn}(x), \quad f_y = 2y,$$

so there are no differentiable interior critical points. On the crease $x = 0$,

$$f(0, y) = y^2.$$

Its smallest value is 0 at the origin. Since $|x| \geq 0$ and $y^2 \geq 0$ everywhere,

$$\boxed{f_{\min} = 0 \text{ at } (0, 0)}.$$



Step 2: Circular boundary Let $u = |x| \in [0, 1]$. On $x^2 + y^2 = 1$,

$$f = u + 1 - u^2 = \frac{5}{4} - \left(u - \frac{1}{2}\right)^2.$$

Thus the largest boundary value is $5/4$, attained when $|x| = 1/2$. Then

$$y^2 = 1 - \frac{1}{4} = \frac{3}{4},$$

so $y = \pm\sqrt{3}/2$.

Step 3: Final comparison For a fixed x , increasing y^2 increases f , so an absolute maximum must lie on the circle; the boundary calculation is therefore decisive. Hence

$$\boxed{f_{\max} = \frac{5}{4} \text{ at } \left(\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)},$$

where the two signs are independent, giving four points. Direct substitution returns $1/2 + 3/4 = 5/4$ at each.