

Question 9

Let R be the closed region between

$$y = x^2 \quad \text{and} \quad y = 2 - x.$$

Find the absolute extrema of

$$f(x, y) = x^2 + y$$

on R .

Tasks

1. Find the boundary intersection points and describe the compact region.
2. Check the interior and each curved boundary piece.
3. Compare all candidates and verify the result from the geometry of the region.

Question 9 – Solution

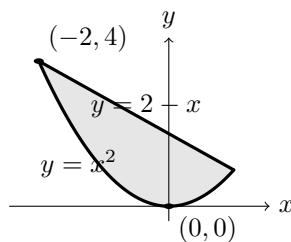
Strategy. Determine the finite interval where the line lies above the parabola, then reduce each boundary to one variable.

Step 1: Region The boundaries meet when

$$x^2 = 2 - x,$$

so $(x + 2)(x - 1) = 0$. The intersection points are $(-2, 4)$ and $(1, 1)$, and

$$-2 \leq x \leq 1, \quad x^2 \leq y \leq 2 - x.$$



Step 2: Candidate search Since $f_y = 1$, there are no interior critical points. On the lower boundary $y = x^2$,

$$f = 2x^2,$$

whose minimum is 0 at $x = 0$ and whose largest endpoint value is 8 at $x = -2$. On the upper boundary $y = 2 - x$,

$$f = x^2 - x + 2 = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4}.$$

Its boundary minimum is $7/4$ at $x = 1/2$; its endpoint values are 8 and 2.

Step 3: Comparison Therefore

$$\boxed{f_{\min} = 0 \text{ at } (0, 0)}, \quad \boxed{f_{\max} = 8 \text{ at } (-2, 4)}.$$

Because $f = x^2 + y$ increases with y , it is also geometrically consistent that vertical comparisons reduce to the lower curve for the minimum and upper curve for the maximum.