

Question 5

Find the absolute extrema of

$$f(x, y) = x^2 + y^2 - 3x$$

on the closed annulus

$$1 \leq x^2 + y^2 \leq 4.$$

Tasks

1. Find all interior critical points that belong to the annulus.
2. Analyze both circular boundary components.
3. Compare every candidate and identify the absolute extrema.

Question 5 – Solution

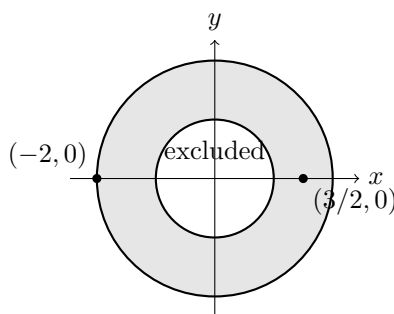
Strategy. Check the annulus interior, then write each boundary circle as $x = r \cos \theta$, $y = r \sin \theta$ with fixed r .

Step 1: Interior

$$f_x = 2x - 3, \quad f_y = 2y.$$

The critical point is $(3/2, 0)$, whose radius $3/2$ lies strictly between 1 and 2. Its value is

$$f(3/2, 0) = \frac{9}{4} - \frac{9}{2} = \boxed{-\frac{9}{4}}.$$



Step 2: Circular boundaries On a circle of radius r ,

$$f = r^2 - 3r \cos \theta.$$

For $r = 1$, its range is $[-2, 4]$. For $r = 2$, its range is $[-2, 10]$. The value 10 occurs on the outer circle when $\cos \theta = -1$, at $(-2, 0)$.

Step 3: Comparison Comparing $-9/4$, the boundary minimum -2 , and the boundary maximum 10 gives

$$\boxed{f_{\min} = -\frac{9}{4} \text{ at } (3/2, 0)}, \quad \boxed{f_{\max} = 10 \text{ at } (-2, 0)}.$$

Both boundary components were checked, so the hole in the region causes no missing candidates.