

Question 3

On the closed triangle

$$T = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 4\},$$

find the absolute extrema of

$$f(x, y) = xy(4 - x - y).$$

Tasks

1. Find every interior critical point.
2. Analyze all three edges and vertices.
3. Compare the candidates and interpret why the minimum occurs along an entire boundary.

Question 3 – Solution

Strategy. The compact triangle guarantees extrema; compare the interior stationary value with the three edge restrictions.

Step 1: Interior

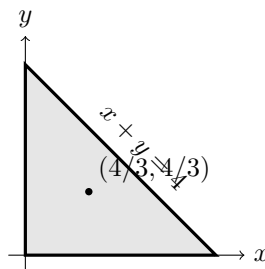
$$f_x = y(4 - 2x - y), \quad f_y = x(4 - x - 2y).$$

In the interior $x, y > 0$, so

$$2x + y = 4, \quad x + 2y = 4.$$

Thus $x = y = 4/3$, and

$$f\left(\frac{4}{3}, \frac{4}{3}\right) = \frac{4}{3} \frac{4}{3} \frac{4}{3} = \boxed{\frac{64}{27}}.$$



Step 2: Boundary On $x = 0$ or $y = 0$, one factor vanishes. On $x + y = 4$, the third factor vanishes. Hence

$$f = 0 \quad \text{on all three edges, including the vertices.}$$

Step 3: Comparison Inside T , all three factors x , y , and $4 - x - y$ are positive, so $f > 0$. Therefore

$$\boxed{f_{\min} = 0 \text{ at every boundary point of } T},$$

$$\boxed{f_{\max} = \frac{64}{27} \text{ at } (4/3, 4/3)}.$$

The candidate list is complete because the polynomial is continuous and T is closed and bounded.