

Question 2

Find the absolute extrema of

$$f(x, y) = x^2 + y^2 - 2x$$

on the closed unit disk $D : x^2 + y^2 \leq 1$.

Tasks

1. Check for interior critical points.
2. Parameterize the circular boundary and find all boundary candidates.
3. Compare the candidates and verify the result geometrically by completing the square.

Question 2 – Solution

Strategy. Treat the interior and the circular boundary separately; on the boundary the radial term is constant.

Step 1: Interior

$$f_x = 2x - 2, \quad f_y = 2y.$$

The only solution is $(1, 0)$, which lies on the boundary rather than in the interior. Thus there are no interior critical points.

Step 2: Boundary Set $x = \cos \theta$, $y = \sin \theta$. Then

$$f(\cos \theta, \sin \theta) = 1 - 2 \cos \theta.$$

The smallest value occurs when $\cos \theta = 1$ and the largest when $\cos \theta = -1$. Therefore

$$\boxed{f_{\min} = -1 \text{ at } (1, 0)}, \quad \boxed{f_{\max} = 3 \text{ at } (-1, 0)}.$$

Step 3: Geometric verification

$$f(x, y) = (x - 1)^2 + y^2 - 1.$$

Thus $f + 1$ is squared distance from (x, y) to $(1, 0)$. Within the disk, that distance is smallest at $(1, 0)$ and largest at the diametrically opposite point $(-1, 0)$. This confirms both locations and values.