

**Question 1**

Find the absolute extrema of

$$f(x, y) = x^2 + y^2 - 2x - 4y$$

on the rectangle  $R = [0, 3] \times [0, 4]$ .

**Tasks**

1. Find all interior critical points.
2. Analyze all four boundary segments, including their endpoints.
3. Compare the complete candidate list and report every absolute extremum.

**Question 1 – Solution**

**Strategy.** On a closed, bounded rectangle, compare values at interior critical points, one-variable boundary critical points, and corners.

**Step 1: Interior**

$$f_x = 2x - 2, \quad f_y = 2y - 4.$$

The sole interior critical point is  $(1, 2)$ , where

$$f(1, 2) = 1 + 4 - 2 - 8 = \boxed{-5}.$$

**Step 2: Boundaries** On  $x = 0$  and  $x = 3$ ,

$$f(0, y) = y^2 - 4y, \quad f(3, y) = y^2 - 4y + 3.$$

Both have a boundary critical value at  $y = 2$ , giving  $-4$  and  $-1$ . On  $y = 0$  and  $y = 4$ ,

$$f(x, 0) = f(x, 4) = x^2 - 2x,$$

whose critical point  $x = 1$  gives  $-1$ . The four corner values are

$$f(0, 0) = 0, \quad f(3, 0) = 3, \quad f(0, 4) = 0, \quad f(3, 4) = 3.$$

**Step 3: Comparison** The smallest candidate value is  $-5$  and the largest is  $3$ . Hence

$$\boxed{f_{\min} = -5 \text{ at } (1, 2)},$$

$$\boxed{f_{\max} = 3 \text{ at } (3, 0) \text{ and } (3, 4)}.$$

Completing the square,  $f = (x - 1)^2 + (y - 2)^2 - 5$ , independently confirms the comparison.