

Question 6

Consider the parameterized quadratic

$$f_{a,b}(x, y) = x^2 + axy + by^2 - 4x + 2y.$$

The point $P = (1, -1)$ is known to be critical.

Tasks

1. Determine a and b from the critical-point condition.
2. Classify P for the resulting function.
3. Verify the classification with two lines through P .

Question 6 – Solution

Strategy. Use the two first-derivative equations to reconstruct the parameters, then test the resulting quadratic form around P .

Step 1: Determine the parameters

$$f_x = 2x + ay - 4, \quad f_y = ax + 2by + 2.$$

At $(1, -1)$, the equations $f_x = f_y = 0$ become

$$-2 - a = 0, \quad a - 2b + 2 = 0.$$

Therefore

$$\boxed{a = -2, \quad b = 0}.$$

Step 2: Hessian classification For the resulting function,

$$f_{xx} = 2, \quad f_{yy} = 0, \quad f_{xy} = -2.$$

Thus

$$D = (2)(0) - (-2)^2 = -4 < 0,$$

so P is a saddle point.

Step 3: Direct verification Write $x = 1 + h$, $y = -1 + k$. Since the function is quadratic and P is critical,

$$f(1 + h, -1 + k) - f(1, -1) = h^2 - 2hk.$$

Along $k = 0$, the change is $h^2 > 0$. Along $k = h$, it is $-h^2 < 0$. Hence every neighborhood of P contains values above and below $f(P)$, confirming

$$\boxed{P = (1, -1) \text{ is a saddle point.}}$$