

Question 8

For the level surface

$$xyz = 8,$$

let $P = (2, 2, 2)$.

Tasks

1. Find the tangent plane and normal line at P .
2. Show that the normal line passes through the origin.
3. Prove that P is the perpendicular foot from the origin to the tangent plane and find that distance in two ways.

Question 8 – Solution

Strategy. Use the gradient for the plane and line, then compare the point-to-plane formula with the length of the normal segment.

Step 1: Plane and line For $F = xyz$,

$$\nabla F = \langle yz, xz, xy \rangle, \quad \nabla F(P) = \langle 4, 4, 4 \rangle.$$

Thus the tangent plane is

$$4(x - 2) + 4(y - 2) + 4(z - 2) = 0,$$

or

$$\boxed{x + y + z = 6}.$$

A normal line is

$$\boxed{(x, y, z) = (2, 2, 2) + t(1, 1, 1)}.$$

Step 2: Origin on the line At $t = -2$, all three coordinates are zero. Thus the segment from the origin to P follows the plane's normal direction, making P the perpendicular foot.

Step 3: Distance checks The normal-segment length is

$$\| \langle 2, 2, 2 \rangle \| = \boxed{2\sqrt{3}}.$$

The point-to-plane formula independently gives

$$\frac{|0 + 0 + 0 - 6|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{6}{\sqrt{3}} = \boxed{2\sqrt{3}}.$$

The agreement verifies both the geometry and the scale of the normal vector.