

Question 7

An unknown member of the family

$$F(x, y, z) = x^2 + ay^2 + bz^2 = c$$

passes through $P = (1, 1, 1)$ and has tangent plane

$$x + 2y + 3z = 6$$

there.

Tasks

1. Determine a , b , and c .
2. Confirm that P is regular for the resulting surface.
3. Write its normal line at P and verify the prescribed tangent plane.

Question 7 – Solution

Strategy. Match the unknown gradient to the plane's normal; the first gradient component fixes the proportionality factor.

Step 1: Match normals

$$\nabla F(P) = \langle 2, 2a, 2b \rangle.$$

This must equal $\lambda \langle 1, 2, 3 \rangle$. The first component gives $\lambda = 2$. Therefore

$$2a = 4, \quad 2b = 6,$$

so

$$\boxed{a = 2, \quad b = 3}.$$

Since P lies on the surface,

$$c = 1^2 + 2(1)^2 + 3(1)^2 = \boxed{6}.$$

Step 2: Regularity For the resulting surface,

$$\nabla F(P) = \langle 2, 4, 6 \rangle \neq \langle 0, 0, 0 \rangle,$$

so P is regular.

Step 3: Line and plane verification The normal line is

$$\boxed{(x, y, z) = (1, 1, 1) + t(1, 2, 3)}.$$

The plane obtained from the gradient is

$$2(x - 1) + 4(y - 1) + 6(z - 1) = 0.$$

Dividing by 2 produces $x + 2y + 3z = 6$, exactly the prescribed plane.