

Question 3

The sphere

$$x^2 + y^2 + z^2 = 9$$

and the plane

$$x + y + z = 5$$

intersect in a curve through $P = (1, 2, 2)$.

Tasks

1. Find a direction tangent to the intersection curve at P .
2. Write the tangent line to the curve.
3. Verify that the direction is tangent to both surfaces.

Question 3 – Solution

Strategy. A curve lying on both surfaces has a tangent perpendicular to both surface normals, so use their cross product.

Step 1: Surface normals Let

$$F = x^2 + y^2 + z^2, \quad G = x + y + z.$$

At P ,

$$\nabla F(P) = \langle 2, 4, 4 \rangle, \quad \nabla G(P) = \langle 1, 1, 1 \rangle.$$

Their cross product is

$$\nabla F(P) \times \nabla G(P) = \langle 0, 2, -2 \rangle,$$

so a simplified tangent direction is $\langle 0, 1, -1 \rangle$.

Step 2: Tangent line

$$\boxed{(x, y, z) = (1, 2, 2) + t(0, 1, -1)}.$$

Step 3: Verification First, $1^2 + 2^2 + 2^2 = 9$ and $1 + 2 + 2 = 5$, so P lies on both surfaces. Next,

$$\langle 2, 4, 4 \rangle \cdot \langle 0, 1, -1 \rangle = 0, \quad \langle 1, 1, 1 \rangle \cdot \langle 0, 1, -1 \rangle = 0.$$

Thus the direction lies in both tangent planes, as required.