

Question 2

Let the level surface

$$F(x, y, z) = xye^z = 2$$

contain $P = (1, 2, 0)$.

Tasks

1. Find the two unit normal vectors at P .
2. Write the tangent plane in standard form.
3. Give symmetric or parametric equations for the normal line.

Question 2 – Solution

Strategy. Differentiate the defining level function; its gradient supplies both the plane normal and the normal-line direction.

Step 1: Gradient

$$\nabla F = \langle ye^z, xe^z, xye^z \rangle.$$

At $P = (1, 2, 0)$,

$$\nabla F(P) = \langle 2, 1, 2 \rangle, \quad \|\nabla F(P)\| = 3.$$

Therefore the two unit normals are

$$\pm \frac{1}{3} \langle 2, 1, 2 \rangle.$$

Step 2: Tangent plane

$$2(x - 1) + (y - 2) + 2(z - 0) = 0,$$

so

$$2x + y + 2z = 4.$$

Step 3: Normal line Using the unsimplified integer direction,

$$(x, y, z) = (1, 2, 0) + t(2, 1, 2).$$

Equivalently,

$$\frac{x - 1}{2} = y - 2 = \frac{z}{2}.$$

Verification The point satisfies $1 \cdot 2 \cdot e^0 = 2$. The line direction equals the plane's coefficient vector, so it is perpendicular to the plane.