

Question 1

Consider the ellipsoid

$$F(x, y, z) = x^2 + 2y^2 + 3z^2 = 6$$

at $P = (1, 1, 1)$.

Tasks

1. Compute the gradient and verify that P is a regular point.
2. Find the tangent plane at P .
3. Find a parametric equation of the normal line and verify its direction.

Question 1 – Solution

Strategy. For a regular level surface, the gradient at the point is normal to the tangent plane.

Step 1: Gradient and regularity

$$\nabla F = \langle 2x, 4y, 6z \rangle, \quad \nabla F(1, 1, 1) = \langle 2, 4, 6 \rangle.$$

This vector is nonzero, so P is regular. Also $F(1, 1, 1) = 1 + 2 + 3 = 6$, confirming that P lies on the surface.

Step 2: Tangent plane Using $\nabla F(P)$ as a normal,

$$2(x - 1) + 4(y - 1) + 6(z - 1) = 0.$$

After division by 2,

$$\boxed{x + 2y + 3z = 6}.$$

Step 3: Normal line A simplified direction parallel to the gradient is $\langle 1, 2, 3 \rangle$. Hence

$$\boxed{(x, y, z) = (1, 1, 1) + t(1, 2, 3)},$$

or $x = 1 + t$, $y = 1 + 2t$, $z = 1 + 3t$.

Verification The line passes through P at $t = 0$, and $\langle 1, 2, 3 \rangle$ is perpendicular to every displacement $\langle u, v, w \rangle$ in the plane because the plane equation requires $u + 2v + 3w = 0$.