

Question 9

A cylinder has volume

$$V(r, h) = \pi r^2 h.$$

Its nominal dimensions are $r = 10$ cm and $h = 20$ cm. A measured cylinder instead has $\Delta r = 0.03$ cm and $\Delta h = -0.08$ cm.

Tasks

1. Use linearization to estimate the change in volume.
2. Estimate the relative and percentage changes.
3. Compute the exact change and quantify the linearization error.

Question 9 – Solution

Strategy. Evaluate the tangent-plane increment $\Delta V \approx V_r \Delta r + V_h \Delta h$ at the nominal dimensions.

Step 1: Volume change

$$V_r = 2\pi r h, \quad V_h = \pi r^2.$$

At $(10, 20)$,

$$dV = (400\pi)(0.03) + (100\pi)(-0.08) = 12\pi - 8\pi.$$

Thus

$$\boxed{\Delta V \approx 4\pi \text{ cm}^3 \approx 12.566 \text{ cm}^3}.$$

Step 2: Relative change Dividing the differential by $V = \pi r^2 h$ gives

$$\frac{dV}{V} = 2\frac{dr}{r} + \frac{dh}{h} = 2\frac{0.03}{10} - \frac{0.08}{20} = 0.002.$$

The estimated percentage increase is $\boxed{0.2\%}$.

Step 3: Exact comparison

$$\Delta V_{\text{exact}} = \pi(10.03)^2(19.92) - 2000\pi = \boxed{3.969928\pi \text{ cm}^3}.$$

The linear estimate exceeds the exact change by

$$(4 - 3.969928)\pi = \boxed{0.030072\pi \text{ cm}^3 \approx 0.09447 \text{ cm}^3}.$$