

Question 7

For the paraboloid

$$z = x^2 + 2y^2,$$

find the tangent plane parallel to the plane

$$2x - 4y + z = 7.$$

Tasks

1. Determine the point of tangency.
2. Write the tangent plane in standard form.
3. Find the perpendicular distance between the two parallel planes.

Question 7 – Solution

Strategy. Match the x - and y -slope coefficients of the tangent plane to those of the given plane.

Step 1: Match slopes The given plane is $z = 7 - 2x + 4y$, so its slopes are -2 and 4 . For $f = x^2 + 2y^2$,

$$f_x = 2x, \quad f_y = 4y.$$

Thus $2x = -2$ and $4y = 4$, giving $(x, y) = (-1, 1)$. The height is $z = 1 + 2 = 3$.

Step 2: Tangent plane At $(-1, 1, 3)$,

$$z - 3 = -2(x + 1) + 4(y - 1),$$

or

$$\boxed{2x - 4y + z = -3}.$$



Step 3: Separation For parallel planes $2x - 4y + z = 7$ and $2x - 4y + z = -3$, the distance is

$$\boxed{\frac{|7 - (-3)|}{\sqrt{2^2 + (-4)^2 + 1^2}} = \frac{10}{\sqrt{21}}}.$$

The identical left sides verify parallelism, and substituting $(-1, 1, 3)$ into the tangent plane gives -3 .