

Question 4

For the graph

$$z = f(x, y) = x^3 - 3x + y^2 - 4y,$$

locate every point at which the tangent plane is horizontal.

Tasks

1. Translate “horizontal tangent plane” into conditions on the partial derivatives.
2. Find all corresponding points on the graph.
3. Write each horizontal tangent plane and show that no others exist.

Question 4 – Solution

Strategy. A tangent plane to a graph is horizontal precisely when both of its slope coefficients vanish.

Step 1: Slope conditions

$$f_x = 3x^2 - 3, \quad f_y = 2y - 4.$$

Thus a horizontal plane requires

$$3x^2 - 3 = 0, \quad 2y - 4 = 0.$$

Therefore $x = \pm 1$ and $y = 2$.

Step 2: Heights

$$f(1, 2) = 1 - 3 + 4 - 8 = -6,$$

$$f(-1, 2) = -1 + 3 + 4 - 8 = -2.$$

The two points are

$$\boxed{(1, 2, -6) \quad \text{and} \quad (-1, 2, -2)}.$$

Step 3: Planes and completeness Because both slope coefficients are zero, the planes are

$$\boxed{z = -6} \quad \text{and} \quad \boxed{z = -2}.$$

The equations $x^2 = 1$ and $y = 2$ exhaust every simultaneous zero of f_x and f_y , so there are no other horizontal tangent planes.