

Question 3

A differentiable function f has tangent plane

$$z = 7 + 3(x - 2) - 4(y + 1)$$

at the point whose input coordinates are $(2, -1)$.

Tasks

1. Recover $f(2, -1)$, $f_x(2, -1)$, and $f_y(2, -1)$.
2. Predict the first-order change along $x = 2 + t$, $y = -1 + 2t$.
3. Construct one nonlinear polynomial f having exactly this tangent plane.

Question 3 – Solution

Strategy. Read the constant and slope coefficients from the standard linearization form, then add terms that vanish to first order.

Step 1: Recover the data Comparing with

$$L = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

at $(a, b) = (2, -1)$ gives

$$\boxed{f(2, -1) = 7}, \quad \boxed{f_x(2, -1) = 3}, \quad \boxed{f_y(2, -1) = -4}.$$

Step 2: Change along the path The input changes are $\Delta x = t$ and $\Delta y = 2t$. Hence

$$\Delta f \approx 3t - 4(2t) = \boxed{-5t}.$$

Step 3: Construct an example One choice is

$$\boxed{f(x, y) = 7 + 3(x - 2) - 4(y + 1) + (x - 2)^2 + (y + 1)^2}.$$

The two quadratic terms and their first partial derivatives vanish at $(2, -1)$, so they do not alter the tangent plane.

Verification Substitution gives $f(2, -1) = 7$. Differentiation gives $f_x = 3 + 2(x - 2)$ and $f_y = -4 + 2(y + 1)$, which reproduce the recovered slopes at the base point.