

Question 2

The upper hemisphere

$$z = \sqrt{25 - x^2 - y^2}$$

is sampled near the point $(3, 0, 4)$.

Tasks

1. Derive the linearization at $(x, y) = (3, 0)$.
2. Estimate the height above $(3.04, 0.06)$.
3. Explain geometrically why the first-order estimate has no y -correction.

Question 2 – Solution

Strategy. Differentiate the height function, then evaluate its first-order change from the base point.

Step 1: Partial derivatives For $f(x, y) = (25 - x^2 - y^2)^{1/2}$,

$$f_x = -\frac{x}{\sqrt{25 - x^2 - y^2}}, \quad f_y = -\frac{y}{\sqrt{25 - x^2 - y^2}}.$$

At $(3, 0)$, these become $f_x = -3/4$ and $f_y = 0$.

Step 2: Linearization

$$L(x, y) = 4 - \frac{3}{4}(x - 3).$$

At the requested point,

$$f(3.04, 0.06) \approx 4 - \frac{3}{4}(0.04) = \boxed{3.9700}.$$

Step 3: Geometry The sphere is symmetric under $y \mapsto -y$. At $y = 0$, moving a small distance in either y -direction changes the height only to second order, so the tangent plane has no y -tilt.

Verification The exact height is

$$\sqrt{25 - 3.04^2 - 0.06^2} = \sqrt{15.7548} \approx 3.9692317.$$

The estimate is high by about 0.0007683.