

**Question 1**

For the graph

$$z = f(x, y) = x^2 e^y,$$

consider the point above  $(1, 0)$ .

**Tasks**

1. Find the tangent plane and the linearization  $L(x, y)$  at  $(1, 0)$ .
2. Use  $L$  to estimate  $f(1.02, -0.03)$ .
3. Compare the estimate with the calculator value  $1.0404e^{-0.03}$ .

**Question 1 – Solution**

**Strategy.** Use the value and both first partial derivatives at the base point to build the unique affine approximation.

**Step 1: Local data**

$$f(1, 0) = 1, \quad f_x = 2xe^y, \quad f_y = x^2e^y.$$

Thus  $f_x(1, 0) = 2$  and  $f_y(1, 0) = 1$ .

**Step 2: Plane and linearization** The tangent-plane formula gives

$$z - 1 = 2(x - 1) + (y - 0).$$

Consequently,

$$\boxed{z = 1 + 2(x - 1) + y}, \quad \boxed{L(x, y) = 1 + 2(x - 1) + y}.$$

**Step 3: Estimate** Here  $\Delta x = 0.02$  and  $\Delta y = -0.03$ , so

$$f(1.02, -0.03) \approx L(1.02, -0.03) = 1 + 2(0.02) - 0.03 = \boxed{1.0100}.$$

**Verification** Direct evaluation gives

$$1.0404e^{-0.03} \approx 1.0096515.$$

The linearization is high by about 0.0003485, a small error consistent with the small displacement from  $(1, 0)$ .