

Question 10

At a point P , a differentiable function has unknown gradient. Its directional derivative equals 3 in every unit direction making angle $\pi/3$ with a fixed unit vector a .

Tasks

1. Determine what this implies about the component of $\nabla f(P)$ along a .
2. In two dimensions, use the two such directions to prove the perpendicular component is zero.
3. Determine $\nabla f(P)$ and the maximum directional derivative.

Question 10 – Solution

Strategy. Resolve the gradient into components parallel and perpendicular to a and compare symmetric directions.

Step 1: Decompose Let b be a unit vector perpendicular to a , and write

$$\nabla f = Aa + Bb.$$

The two unit directions at angles $\pm\pi/3$ from a are

$$u_{\pm} = \frac{1}{2}a \pm \frac{\sqrt{3}}{2}b.$$

Step 2: Use both measurements

$$3 = \nabla f \cdot u_{\pm} = \frac{A}{2} \pm \frac{\sqrt{3}B}{2}.$$

Adding gives $6 = A$, while subtracting gives $B = 0$.

Step 3: Result Therefore

$$\boxed{\nabla f(P) = 6a}.$$

The greatest directional derivative is the gradient magnitude,

$$\boxed{D_{\max}f(P) = 6},$$

attained uniquely in direction a . Checking either prescribed direction gives $6 \cos(\pi/3) = 3$.