

Question 8

At $P = (1, 2)$, an unknown differentiable function satisfies

$$D_{u_1} f(P) = 4, \quad u_1 = \langle 1, 0 \rangle,$$

$$D_{u_2} f(P) = 1, \quad u_2 = \langle 3/5, 4/5 \rangle.$$

Tasks

1. Reconstruct $\nabla f(P)$.
2. Find the steepest-ascent direction and rate.
3. Predict the derivative in direction $\langle 0, -1 \rangle$ and verify consistency.

Question 8 – Solution

Strategy. Treat directional measurements as linear equations for the two gradient components.

Step 1: Reconstruct Write $\nabla f(P) = \langle a, b \rangle$. The first measurement gives $a = 4$. The second gives

$$\frac{3}{5}a + \frac{4}{5}b = 1.$$

Thus $12/5 + 4b/5 = 1$, so $b = -7/4$ and

$$\boxed{\nabla f(P) = \langle 4, -7/4 \rangle}.$$

Step 2: Maximum

$$\|\nabla f(P)\| = \sqrt{16 + \frac{49}{16}} = \frac{\sqrt{305}}{4}.$$

The steepest-ascent direction is

$$\boxed{\frac{1}{\sqrt{305}} \langle 16, -7 \rangle},$$

and its rate is $\boxed{\sqrt{305}/4}$.

Step 3: New direction For $\langle 0, -1 \rangle$,

$$D_{\langle 0, -1 \rangle} f = \langle 4, -7/4 \rangle \cdot \langle 0, -1 \rangle = \boxed{7/4}.$$

Substitution into both original dot products reproduces 4 and 1.