

Question 6

For the scalar field

$$F(x, y, z) = xe^{yz} + z^2,$$

find the directional derivative at $P = (2, 0, 1)$ toward $Q = (-1, 2, 3)$.

Tasks

1. Find the three-dimensional unit direction.
2. Compute $\nabla F(P)$.
3. Evaluate the rate and verify its units conceptually.

Question 6 – Solution

Strategy. The same gradient-dot-unit-vector formula applies in three dimensions.

Step 1: Direction

$$\overrightarrow{PQ} = \langle -3, 2, 2 \rangle, \quad \|\overrightarrow{PQ}\| = \sqrt{17}, \quad u = \frac{1}{\sqrt{17}} \langle -3, 2, 2 \rangle.$$

Step 2: Gradient

$$F_x = e^{yz}, \quad F_y = xze^{yz}, \quad F_z = xye^{yz} + 2z.$$

At $(2, 0, 1)$,

$$\nabla F(P) = \langle 1, 2, 2 \rangle.$$

Step 3: Rate

$$D_u F(P) = \frac{-3 + 4 + 4}{\sqrt{17}} = \boxed{\frac{5}{\sqrt{17}}}.$$

Because u is dimensionless and unit length, the result measures change in F per unit spatial distance.