

Question 5

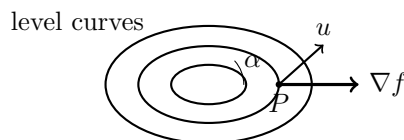
Let f be continuously differentiable near P . At P , its level curve is smooth and $\nabla f(P) \neq 0$. A unit direction u makes angle α with $\nabla f(P)$.

Tasks

1. Derive $D_u f(P)$ in terms of α .
2. Explain why tangent motion along the level curve gives zero rate.
3. Determine the sign of the rate for acute, right, and obtuse α .

Question 5 – Solution

Strategy. Combine the dot-product formula with the gradient's normal relationship to level curves.



Step 1: Angular formula

$$D_u f(P) = \nabla f(P) \cdot u = \|\nabla f(P)\| \cos \alpha.$$

Step 2: Tangent direction Along the level curve, f is constant, so its rate is zero. Geometrically the tangent is perpendicular to ∇f , giving $\alpha = \pi/2$ and $\cos \alpha = 0$.

Step 3: Signs The rate is positive for acute α , zero for a right angle, and negative for obtuse α . Its magnitude is greatest when u is parallel or antiparallel to the gradient.