

Question 4

At P , $\nabla f(P) = \langle 6, 8 \rangle$. Determine every unit direction u for which $D_u f(P) = 5$.

Tasks

1. Convert the condition to geometry.
2. Find both unit vectors exactly.
3. Verify normalization and the required rate.

Question 4 – Solution

Strategy. Decompose the desired unit vector into components parallel and perpendicular to the gradient.

Step 1: Unit basis Let

$$e = \frac{1}{10} \langle 6, 8 \rangle = \langle 3/5, 4/5 \rangle, \quad n = \langle -4/5, 3/5 \rangle.$$

These form an orthonormal basis. Write $u = ae + bn$ with $a^2 + b^2 = 1$.

Step 2: Rate condition

$$\nabla f \cdot u = 10a = 5,$$

so $a = 1/2$ and $b = \pm\sqrt{3}/2$. Thus

$$u = \frac{1}{2} \langle 3/5, 4/5 \rangle \pm \frac{\sqrt{3}}{2} \langle -4/5, 3/5 \rangle.$$

Equivalently,

$$u = \left\langle \frac{3 \mp 4\sqrt{3}}{10}, \frac{4 \pm 3\sqrt{3}}{10} \right\rangle.$$

Step 3: Verify Orthonormality gives $\|u\|^2 = 1/4 + 3/4 = 1$, and the perpendicular component contributes zero to the dot product, leaving $10(1/2) = 5$.