

**Question 3**

At a point  $P$ , a differentiable function has  $\nabla f(P) = \langle -2, 5 \rangle$ .

**Tasks**

1. Find the maximum and minimum directional derivatives and their directions.
2. Find all unit directions giving zero directional derivative.
3. Find the directional derivative at angle  $\pi/6$  from the positive  $x$ -axis.

**Question 3 – Solution**

**Strategy.** Use  $D_u f = \|\nabla f\| \cos \alpha$ , where  $\alpha$  is the angle from the gradient to the unit direction.

**Step 1: Extremes** Since  $\|\nabla f\| = \sqrt{29}$ ,

$$D_{\max} = \sqrt{29} \text{ in direction } \frac{1}{\sqrt{29}} \langle -2, 5 \rangle,$$

$$D_{\min} = -\sqrt{29} \text{ in direction } \frac{1}{\sqrt{29}} \langle 2, -5 \rangle.$$

**Step 2: Zero directions** Unit vectors perpendicular to  $\langle -2, 5 \rangle$  are

$$u = \pm \frac{1}{\sqrt{29}} \langle 5, 2 \rangle.$$

**Step 3: Prescribed angle** For  $u = \langle \sqrt{3}/2, 1/2 \rangle$ ,

$$D_u f = -2 \frac{\sqrt{3}}{2} + 5 \frac{1}{2} = \frac{5}{2} - \sqrt{3}.$$