

Question 2

Let $g(x, y) = \ln(x^2 + y^2)$. Find its rate of change at $P = (1, -2)$ toward $Q = (4, 2)$.

Tasks

1. Construct the unit vector from P to Q .
2. Compute the directional derivative exactly.
3. Decide whether g initially increases or decreases.

Question 2 – Solution

Strategy. Subtract endpoint coordinates in the correct order before normalizing.

Step 1: Direction

$$\overrightarrow{PQ} = \langle 3, 4 \rangle, \quad u = \langle 3/5, 4/5 \rangle.$$

Step 2: Gradient

$$\nabla g = \left\langle \frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2} \right\rangle,$$

so

$$\nabla g(1, -2) = \langle 2/5, -4/5 \rangle.$$

Therefore

$$D_u g(P) = \frac{2}{5} \frac{3}{5} - \frac{4}{5} \frac{4}{5} = \boxed{-\frac{2}{5}}.$$

Step 3: Interpret The negative sign means g decreases initially toward Q . This is consistent with the initial motion having a component toward smaller distance from the origin.