

Question 1

For

$$f(x, y) = x^2y + e^{x-y},$$

find the directional derivative at $P = (1, 0)$ in the direction $v = \langle 3, 4 \rangle$.

Tasks

1. Normalize the direction.
2. Compute the gradient at P .
3. Evaluate and interpret the directional derivative.

Question 1 – Solution

Strategy. A directional derivative uses a unit vector: $D_u f = \nabla f \cdot u$.

Step 1: Normalize

$$\|v\| = 5, \quad u = \boxed{\langle 3/5, 4/5 \rangle}.$$

Step 2: Gradient

$$f_x = 2xy + e^{x-y}, \quad f_y = x^2 - e^{x-y}.$$

At $(1, 0)$,

$$\nabla f(1, 0) = \langle e, 1 - e \rangle.$$

Step 3: Dot product

$$D_u f(1, 0) = e \frac{3}{5} + (1 - e) \frac{4}{5} = \boxed{\frac{4 - e}{5}}.$$

The positive value means f initially increases by approximately $(4 - e)/5$ output units per unit distance traveled in the specified direction.