

Question 9

Variables x, y are defined implicitly by

$$x + y = s, \quad xy = t.$$

Near $(x, y) = (1, 2)$, regard x, y as functions of (s, t) .

Tasks

1. Find x_s, y_s, x_t, y_t by differentiating the system.
2. Evaluate them at $(1, 2)$.
3. Explain why the calculation fails when $x = y$.

Question 9 – Solution

Strategy. Differentiate both constraints and solve the resulting 2×2 linear systems.

Step 1: With respect to s

$$x_s + y_s = 1, \quad yx_s + xy_s = 0.$$

Solving gives

$$\boxed{x_s = \frac{x}{x-y}}, \quad \boxed{y_s = -\frac{y}{x-y}}.$$

Step 2: With respect to t

$$x_t + y_t = 0, \quad yx_t + xy_t = 1,$$

so

$$\boxed{x_t = \frac{1}{y-x}}, \quad \boxed{y_t = \frac{1}{x-y}}.$$

At $(1, 2)$:

$$\boxed{x_s = -1, \ y_s = 2, \ x_t = 1, \ y_t = -1}.$$

Step 3: Edge case The determinant of the coefficient matrix is $x - y$. When $x = y$, the system cannot locally distinguish the two roots; division by $x - y$ correctly signals failure of this local inverse description.