

Question 8

Suppose $z = f(x, y)$ has $f_x(2, 0) = 3$ and $f_y(2, 0) = -4$. The inputs obey

$$x = u^2 + v, \quad y = u - v^2.$$

At $(u, v) = (1, 1)$:

Tasks

1. Find z_u and z_v .
2. Find all input changes (du, dv) producing zero first-order change in z .
3. Verify the cancellation using dx, dy .

Question 8 – Solution

Strategy. First propagate u, v changes to x, y , then combine them with the supplied partials.

Step 1: Chain derivatives At $(1, 1)$, $(x, y) = (2, 0)$ and

$$x_u = 2, \ x_v = 1, \ y_u = 1, \ y_v = -2.$$

Therefore

$$\boxed{z_u = 3(2) + (-4)(1) = 2}, \quad \boxed{z_v = 3(1) + (-4)(-2) = 11}.$$

Step 2: Zero-change inputs The total differential is

$$dz = 2 \, du + 11 \, dv.$$

Thus every first-order neutral change satisfies

$$\boxed{2 \, du + 11 \, dv = 0},$$

equivalently $(du, dv) = \lambda(11, -2)$.

Step 3: Verify For $(du, dv) = (11, -2)$, $dx = 2(11) - 2 = 20$ and $dy = 11 - 2(-2) = 15$. Then $dz = 3(20) - 4(15) = 0$.