

Question 7

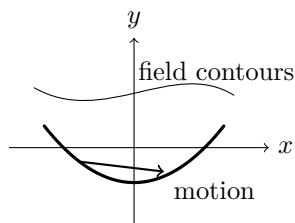
A temperature field is $T(x, y, t) = x^2 + 2xy + 3t$. A particle follows $x = t^2$, $y = 1 - t$.

Tasks

1. Find the rate of temperature change experienced by the particle.
2. Evaluate it at $t = 1$.
3. Separate spatial-motion and explicit-time contributions.

Question 7 – Solution

Strategy. The experienced rate includes motion through space and direct evolution of the field.



Step 1: Extended chain rule

$$\frac{dT}{dt} = T_x x' + T_y y' + T_t.$$

Here

$$T_x = 2x + 2y, \quad T_y = 2x, \quad T_t = 3, \quad x' = 2t, \quad y' = -1.$$

Thus

$$\boxed{\frac{dT}{dt} = (2x + 2y)(2t) - 2x + 3}.$$

Step 2: Evaluate At $t = 1$, $(x, y) = (1, 0)$:

$$\boxed{\frac{dT}{dt} = 2(2) - 2 + 3 = 5}.$$

Step 3: Contributions Motion in x contributes 4, motion in y contributes -2 , and explicit time dependence contributes 3. Their sum is 5 temperature units per time unit.