

Question 6

Let

$$F(x, y) = \begin{pmatrix} x^2 - y \\ xy \end{pmatrix}, \quad G(s, t) = \begin{pmatrix} e^s \cos t \\ e^s \sin t \end{pmatrix}.$$

Tasks

1. Find the Jacobian matrices J_F and J_G .
2. Use matrix chain rule to find $J_{F \circ G}(0, \pi/2)$.
3. Check one entry directly.

Question 6 – Solution

Strategy. Evaluate J_F at $G(s, t)$ before multiplying by J_G .

Step 1: Jacobians

$$J_F = \begin{pmatrix} 2x & -1 \\ y & x \end{pmatrix}, \quad J_G = \begin{pmatrix} e^s \cos t & -e^s \sin t \\ e^s \sin t & e^s \cos t \end{pmatrix}.$$

At $(0, \pi/2)$, $G = (0, 1)$, so

$$J_F(0, 1) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad J_G = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Step 2: Multiply

$$J_{F \circ G}(0, \pi/2) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The composite Jacobian is $-I_2$.

Step 3: Check The first component is $e^{2s} \cos^2 t - e^s \sin t$; its s -derivative at the point is $0 - 1 = -1$, matching the upper-left entry.