

Question 4

A surface satisfies $x^2 + y^2 + z^2 = 14$. Along a path on the surface, $x = t$, $y = t^2$, and $z > 0$ near $t = 1$.

Tasks

1. Find dz/dt implicitly without solving for $z(t)$ first.
2. Evaluate at $t = 1$.
3. Verify using the positive explicit branch.

Question 4 – Solution

Strategy. Differentiate the constraint along the path; its total rate must remain zero.

Step 1: Differentiate

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} + 2z \frac{dz}{dt} = 0,$$

so

$$\boxed{\frac{dz}{dt} = -\frac{xx' + yy'}{z}}.$$

Step 2: Evaluate At $t = 1$, $x = 1$, $y = 1$, $z = \sqrt{14 - 1 - 1} = 2\sqrt{3}$, while $x' = 1$, $y' = 2$. Hence

$$\boxed{\left. \frac{dz}{dt} \right|_1 = -\frac{3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}}.$$

Step 3: Check The positive branch is $z(t) = \sqrt{14 - t^2 - t^4}$. Differentiation gives $z' = (-2t - 4t^3)/(2z)$, which produces the same value at $t = 1$.