

Question 3

Let

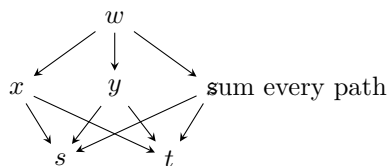
$$w = x^2 + yz, \quad x = s + t, \quad y = st, \quad z = s^2 - t^2.$$

Tasks

1. Draw or use the dependency structure to find w_s, w_t .
2. Evaluate both at $(s, t) = (1, 2)$.
3. Verify one result by explicit expansion.

Question 3 – Solution

Strategy. Sum the contribution along every dependency path.



Step 1: Derivatives

$$w_x = 2x, \quad w_y = z, \quad w_z = y.$$

$$x_s = 1, \quad y_s = t, \quad z_s = 2s; \quad x_t = 1, \quad y_t = s, \quad z_t = -2t.$$

Therefore

$$\boxed{w_s = 2x + zt + 2sy},$$

$$\boxed{w_t = 2x + zs - 2ty}.$$

Step 2: Evaluate At $(1, 2)$, $(x, y, z) = (3, 2, -3)$:

$$\boxed{w_s = 6 - 6 + 4 = 4},$$

$$\boxed{w_t = 6 - 3 - 8 = -5}.$$

Step 3: Check Expanding gives $w = (s+t)^2 + st(s^2 - t^2)$. Differentiating in s yields $2(s+t) + t(3s^2 - t^2)$, which equals 4 at $(1, 2)$.