

Question 2

Suppose $F(x, y) = \ln(x^2 + y^2)$ and a curve is $x = 3 \cos t$, $y = 3 \sin t$.

Tasks

1. Compute dF/dt by the chain rule.
2. Explain the cancellation geometrically.
3. Verify using the explicit composite.

Question 2 – Solution

Strategy. Evaluate the two chain-rule contributions before simplifying.

Step 1: Chain rule

$$F_x = \frac{2x}{x^2 + y^2}, \quad F_y = \frac{2y}{x^2 + y^2},$$
$$x' = -3 \sin t, \quad y' = 3 \cos t.$$

Thus

$$\frac{dF}{dt} = \frac{2x(-3 \sin t) + 2y(3 \cos t)}{x^2 + y^2} = 0,$$

after substituting $x = 3 \cos t$, $y = 3 \sin t$.

Step 2: Meaning The curve remains on the circle $x^2 + y^2 = 9$, and F depends only on that squared radius. Hence $\boxed{dF/dt = 0}$.

Step 3: Direct check $F(3 \cos t, 3 \sin t) = \ln 9$, whose derivative is zero.