

Question 1

Let $z = x^2y + \sin y$, where $x = t^2 - 1$ and $y = e^t$.

Tasks

1. Find dz/dt using the multivariable chain rule.
2. Evaluate it at $t = 0$.
3. Verify by differentiating the explicit composite function.

Question 1 – Solution

Strategy. Combine the two routes $t \rightarrow x \rightarrow z$ and $t \rightarrow y \rightarrow z$.

Step 1: Ingredients

$$z_x = 2xy, \quad z_y = x^2 + \cos y, \quad x' = 2t, \quad y' = e^t.$$

Therefore

$$\frac{dz}{dt} = 2xy(2t) + (x^2 + \cos y)e^t.$$

Step 2: Evaluate At $t = 0$, $x = -1$ and $y = 1$, giving

$$\left. \frac{dz}{dt} \right|_{t=0} = 1 + \cos 1.$$

Step 3: Direct check The composite is

$$z(t) = (t^2 - 1)^2 e^t + \sin(e^t).$$

Its derivative is $4t(t^2 - 1)e^t + (t^2 - 1)^2 e^t + \cos(e^t)e^t$, which at zero is $1 + \cos 1$.