

Question 9

A differential field is proposed as

$$df = (y^2 + 2x) dx + (2xy + e^y) dy, \quad f(0, 0) = 5.$$

Tasks

1. Reconstruct f .
2. Use df to estimate $f(0.02, -0.01)$ from the origin.
3. Compare the estimate with the exact value through second order.

Question 9 – Solution

Strategy. Reconstruct the function first, then separate its linear and nonlinear increment terms.

Step 1: Reconstruct Integrating $f_x = y^2 + 2x$ gives

$$f = xy^2 + x^2 + C(y).$$

Since $f_y = 2xy + C'(y) = 2xy + e^y$, $C = e^y + C_0$. The initial value gives $1 + C_0 = 5$, so

$$\boxed{f = xy^2 + x^2 + e^y + 4}.$$

Step 2: Differential estimate At $(0, 0)$,

$$f_x = 0, \quad f_y = 1.$$

For $dx = 0.02$, $dy = -0.01$,

$$f(0.02, -0.01) \approx 5 + df = 5 - 0.01 = \boxed{4.99}.$$

Step 3: Exact comparison Exactly,

$$f = 0.02(0.01)^2 + (0.02)^2 + e^{-0.01} + 4 \approx 4.99045183.$$

The estimate error is about $\boxed{0.00045183}$, while the second-order correction is $dx^2 + dy^2/2 = 0.00045$. Thus the second-order estimate is 4.99045, consistent with the exact value.