

Question 8

A quantity is

$$Q = \frac{x^3 \sqrt{y}}{z^2}, \quad x, y, z > 0.$$

Tasks

1. Derive a relative-differential formula for dQ/Q .
2. Estimate the signed percentage change if x rises 1%, y falls 4%, and z rises 0.5%.
3. Identify the dominant contribution.

Question 8 – Solution

Strategy. Divide the total differential by Q , or equivalently differentiate the logarithmic power structure.

Step 1: Relative differential

$$\ln Q = 3 \ln x + \frac{1}{2} \ln y - 2 \ln z.$$

Differentiating gives

$$\boxed{\frac{dQ}{Q} = 3 \frac{dx}{x} + \frac{1}{2} \frac{dy}{y} - 2 \frac{dz}{z}}.$$

Step 2: Percentage estimate

$$\frac{dQ}{Q} \approx 3(0.01) + \frac{1}{2}(-0.04) - 2(0.005) = 0.03 - 0.02 - 0.01 = \boxed{0}.$$

Thus the predicted first-order percentage change is $\boxed{0\%}$.

Step 3: Contributions The x increase contributes $+3\%$, the y decrease -2% , and the z increase -1% . The x term is individually largest, but the three first-order effects cancel exactly.