

Question 7

Determine whether

$$\omega = (2xy + 3) dx + (x^2 + 4y) dy$$

is the differential of a scalar function $f(x, y)$.

Tasks

1. Test the necessary compatibility condition.
2. Recover every possible f if the test passes.
3. Verify by redifferentiating.

Question 7 – Solution

Strategy. If $df = M dx + N dy$, then $M_y = N_x$ on a simply connected domain.

Step 1: Compatibility Here

$$M = 2xy + 3, \quad N = x^2 + 4y.$$

Then

$$M_y = 2x = N_x,$$

so the form is compatible throughout $\mathbb{R}^2$.

Step 2: Recover f Integrate $f_x = M$ in x :

$$f = x^2y + 3x + C(y).$$

Then

$$f_y = x^2 + C'(y) = x^2 + 4y,$$

so $C'(y) = 4y$ and $C(y) = 2y^2 + C_0$.

Result and verification

$$\boxed{f(x, y) = x^2y + 3x + 2y^2 + C_0}.$$

Indeed, differentiating gives $df = (2xy + 3) dx + (x^2 + 4y) dy$. The additive constant is invisible to the differential.